

## CONTRIBUTION TO CERENKOV RESOLUTION FROM KNOCK-ON ELECTRONS

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**Abstract:** Calculations of the mean and standard error of the added Cerenkov component from knock-on electrons for sample counters with refractive indices ranging from  $n=1.03$  to  $n=1.49$  are presented. We find that this contribution to the Cerenkov resolution is significant, but not dominant, for typical detector parameters.

**Algorithm:** Knock-on electrons (KOs) produced above and within a Cerenkov radiator by the passage of a high-energy charged particle contribute to the total signal in a Cerenkov counter and to the fluctuations in this signal. Lezniak (1976) developed a formalism to calculate the mean added Cerenkov component from a non-equilibrium distribution of KOs. We have extended his formalism to calculate the standard error in the knock-on component due to fluctuations in the number and energies of the KOs. Lezniak presents his algorithm in detail, and space constraints prevent us from elaborating here; however, some important steps are outlined below. For more detail, especially of the limits of integration below, see Lezniak (1976) and Grove (1989).

First, the number of photoelectrons produced per  $\text{g/cm}^2$  of radiator by a particle of charge  $Z$  and kinetic energy per nucleon  $E$  (or velocity  $\beta(E)$ ) and collected in the viewing system of a Cerenkov counter can be expressed as follows,

$$\frac{dC}{dx}(Z,E) = \frac{Z^2 N_\mu \sec \theta}{t \sec \theta} \frac{1 - 1/n^2 \beta^2}{1 - 1/n^2} \quad \beta > n^{-1}, \quad (1)$$

where  $N_\mu$  is the number of photoelectrons from a vertical primary particle with  $Z=1$  and  $\beta=1$ , and  $n$  is the index of refraction of the radiator. The thickness of the radiator is  $t \text{ g/cm}^2$ , and the particle's angle of incidence is  $\theta$ . Note that the angular dependence cancels.

The number of KOs with energies between  $E'$  and  $E' + dE'$  produced in  $dx' \text{ g/cm}^2$  of target material by a particle of charge  $Z$  at velocity  $\beta$  and Lorentz  $\gamma$  is (e.g., Rossi, 1952)

$$\phi_{\text{mat}}(Z,E,E') dE' dx' = \frac{\alpha_{\text{mat}} Z^2}{\beta^2} \frac{1}{E'^2} \left[ 1 - \beta^2 \frac{E'}{E_{\text{max}}} \right] dE' dx' \quad 0 \leq E' \leq E_{\text{max}} \quad (2)$$

where  $E_{\text{max}} = 2m_e c^2 (\gamma^2 - 1)$  is the maximum energy that can be imparted to a KO, and  $\alpha_{\text{mat}} = 0.30058 m_e c^2 Z_t / A_t \text{ MeV cm}^2/\text{g}$ , where  $Z_t$  and  $A_t$  are the average charge and mass numbers of the target.

We take the detector elements to be infinite in lateral extent. KOs are produced in the overlying material and in the radiator and are allowed to propagate through the detector system, losing energy as they travel. They generate Cerenkov light whenever they are in the radiator and have  $\beta' > n^{-1}$ .

To account for the energy loss of the KOs, we have adopted the extrapolated (or "practical") range relation for electrons from Kobetich and Katz (1968) for aluminum, modified by a multiplicative correction factor  $h(Z_t, A_t)$  to account for the dependence on the target material. From the calculated electron ranges of Berger and Seltzer (1964), we have deduced the following relation for materials free of hydrogen:  $h(Z_t, A_t) \approx 0.911 + 2.41 (1 - 2Z_t/A_t)$ . Hydrogenic compounds do not obey this relation; however, from Berger and Seltzer we deduce that  $h(\text{Lucite}) = 0.813$ . The extrapolated range is calculated from the measured transmission probability of monoenergetic beams of electrons through absorbers of known thickness, and it therefore includes the effects of electron scattering as an average over many beam particles. We have assumed that the extrapolated range is an

adequate estimate of both the pathlength and the average range of electrons in the detector materials.

The Cerenkov signal  $Y(E, \theta, E', x')$  generated by a single KO of kinetic energy  $E'$  produced at position  $x'$  by a primary particle of energy  $E$  is

$$Y(E, \theta, E', x') = \int_{\Delta E} \frac{dC}{dx}(Z=1, E') \frac{dx}{dE}(E') dE' , \quad (3)$$

where the limits of integration ensure that only those KOs with energies above the Cerenkov threshold energy in the radiator are counted.

The average number of photoelectrons  $K(Z, E, \theta)$  generated by all KOs produced by a primary particle is the expectation value of the light yield  $Y(E, \theta, E', x')$  per KO multiplied by the number of KOs  $N_{\text{mat}}(Z, E, \theta)$  that generate Cerenkov radiation:

$$K(Z, E, \theta) = \sum_{\text{mat}} N_{\text{mat}}(Z, E, \theta) \langle Y(E, \theta, E', x') \rangle_{\text{mat}} \quad (4)$$

$$= \sum_{\text{mat}} \int \int \phi_{\text{mat}}(Z, E, E') Y(E, \theta, E', x') dE' dx' \propto Z^2 N_{\mu} \sec \theta . \quad (5)$$

Extending Lezniak's algorithm to calculate the standard error, we assume that the variation in the average Cerenkov yield from all KOs produced by a primary particle is normally distributed, with rms  $\sigma_K(Z, E, \theta)$ . A simple Monte Carlo simulation of the production and propagation of the KOs (Grove, 1989) confirms that this is an adequate assumption for typical detector configurations, although it is poor for the  $n=1.03$  case discussed below.

The variation in the KO contribution to the Cerenkov signal results from fluctuations in both the number of electrons produced and the energies at which they are produced. Propagation of errors in Equation (4) shows that the square of the standard error of the number of photoelectrons from KOs produced in either the radiator or the overlying material is

$$\sigma_{\text{mat}}^2(Z, E, \theta) = N_{\text{mat}}^2(Z, E, \theta) \sigma_Y^2 + \langle Y(E, \theta, E', x') \rangle_{\text{mat}}^2 \sigma_N^2 , \quad (6)$$

where the variance of the mean number of KOs produced is  $\sigma_N^2 = N_{\text{mat}}(Z, E, \theta)$ , and the variance of the mean light yield per KO is

$$\sigma_Y^2 = N_{\text{mat}}^{-1}(Z, E, \theta) \left[ \langle Y^2(E, \theta, E', x') \rangle_{\text{mat}} - \langle Y(E, \theta, E', x') \rangle_{\text{mat}}^2 \right] . \quad (7)$$

Therefore the square of the total standard error is

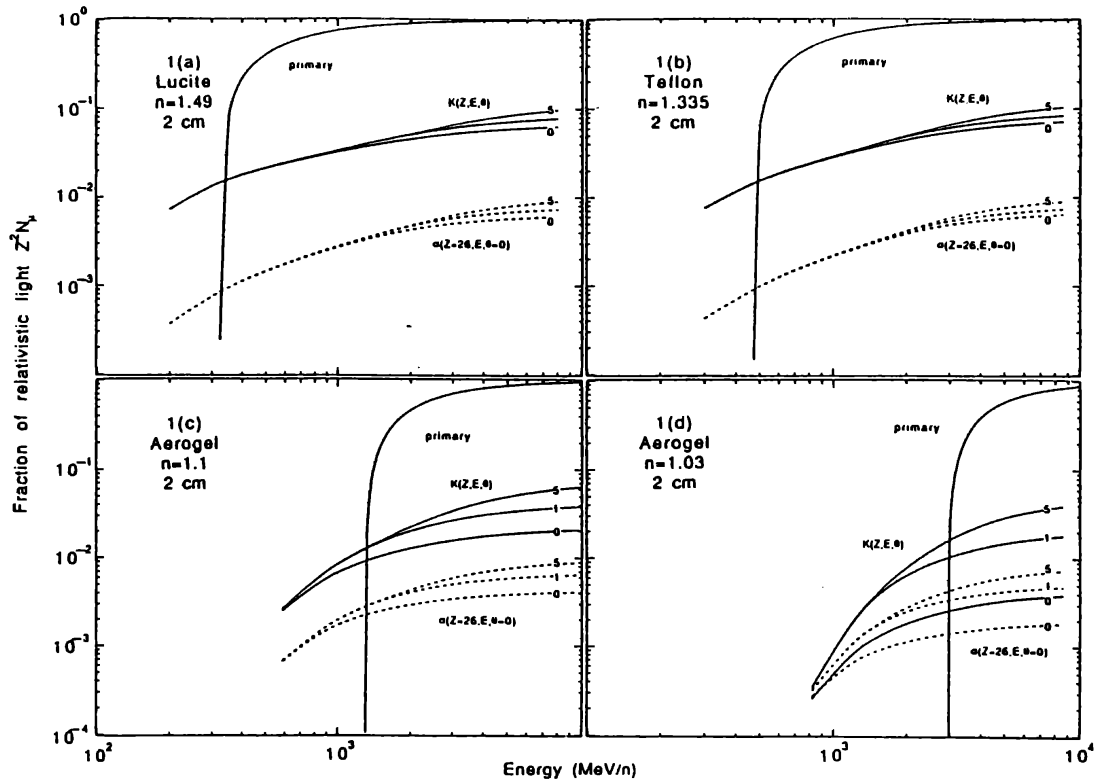
$$\sigma_K^2(Z, E, \theta) = \sum_{\text{mat}} N_{\text{mat}}(Z, E, \theta) \langle Y^2(E, \theta, E', x') \rangle_{\text{mat}} \quad (8)$$

$$= \sum_{\text{mat}} \int \int \phi_{\text{mat}}(Z, E, E') Y^2(E, \theta, E', x') dE' dx' \propto Z^2 N_{\mu}^2 (\sec \theta)^{(1+\epsilon(E))} . \quad (9)$$

The limits of integration can be found in Grove (1989) or Lezniak (1976). The magnitude and functional form of the pathlength correction depend on the parameters of the radiator, so a general expression cannot be given. For the Lucite and Teflon examples discussed below,  $\epsilon(E) < 0.1$  over the useful energy range of the radiator.

**Discussion:** We have modeled four sample radiators: Lucite ( $n=1.49$ ), Teflon ( $n=1.335$ ), sintered silica aerogel ( $n=1.10$ ), and silica aerogel ( $n=1.03$ ). Each radiator is assumed to be 2 cm thick. The results of the calculations are shown in Figures 1(a)-(d) for a vertically incident beam of Fe ( $Z=26$ ) as a function of the beam energy in the middle of the radiator for zero, 1.0, and 5.0 g/cm<sup>2</sup> of overlying material. In figures 2(a)-(d), the radiators were assumed to be positioned in a strong magnetic field. For this condition, we have made the simplifying assumptions that KOs generated in the radiator cannot escape and that the overlying material has no effect.

The Lucite counter (Figures 1(a) and 2(a)) is similar to that used by Lezniak, and the difference in the mean KO component is due primarily to our different choice of the value of



Figures 1(a)-(d): Cerenkov yield of Fe nucleus at  $\theta = 0^\circ$  and its associated KOs as a function of the energy of the primary in the center of the radiator. Curve labeled "primary" is the Cerenkov yield of the primary particle, expressed as a fraction of its yield  $Z^2 N_\mu$  at velocity  $\beta = 1$ . The mean KO contribution  $K(Z, E, \theta)$  is shown by the solid lines, and the standard error  $\sigma_K(Z=26, E, \theta=0)$  by the dashed lines. Numbers 0, 1, and 5 indicate thickness of overlying material in  $\text{g}/\text{cm}^2$ .

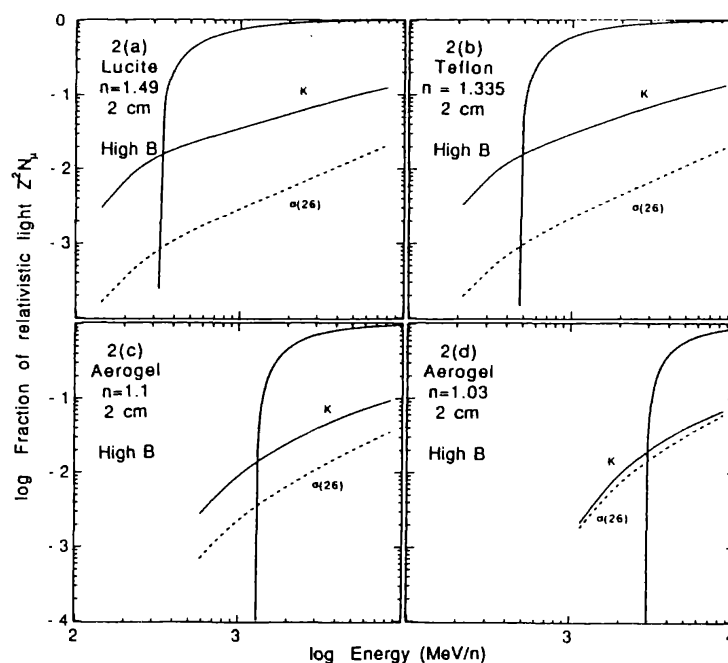
$dE/dx$  for electrons. The  $n = 1.03$  aerogel counter (Figures 1(d) and 2(d)) is similar to one proposed for Astromag (e.g., Ormes *et al.* 1986). Because  $N_{\text{rad}}(Z=26, E, \theta) \sim 6$  in the  $n = 1.03$  counter, the Gaussian assumption is poor: large KO signals are more probable than  $\sigma_K(Z, E, \theta)$  indicates. Note, however, that the calculated mean and standard error remain accurate estimates of the result of the Monte Carlo calculation.

In all four counters in the  $B=0$  case, the mean KO component at the threshold energy is  $\sim 1\%$  of the relativistic primary Cerenkov signal,  $Z^2 N_\mu$ , and the standard error is  $\sim 0.1\%$  for Fe, or in general  $\sim (3/Z)\%$  of  $Z^2 N_\mu$ . In the strong-field case, the mean and standard error are larger (particularly for the  $n = 1.03$  and  $n = 1.1$  examples) because even the highest energy electrons are allowed to radiate over their entire pathlength above the threshold energy.

Estimates for other charges or detector thicknesses can be derived by scaling with Equations (5) and (9): thus the mean KO contribution is approximately independent of  $Z$  and thickness, while the standard error as a fraction of the relativistic light scales approximately as  $Z^{-1}$  and  $(\text{thickness})^{-1}$ , where errors in the latter scaling are  $< 10\%$  for  $n = 1.49$  and  $n = 1.335$ , and  $< 20\%$  for  $n = 1.1$ .

Note that the square of the fractional standard error can be written as

$$\frac{\sigma_K^2(Z, E, \theta)}{K^2(Z, E, \theta)} = \frac{\sigma_Y^2}{\langle Y(E, \theta, E', x')^2 \rangle N_{\text{mat}}(Z, E, \theta)} + \frac{1}{N_{\text{mat}}(Z, E, \theta)}, \quad (10)$$



Figures 2(a)-(d): Same as Figure 1, except radiators assumed to be positioned in strong B field. KOs are not permitted to escape from radiator, and overlying material is ignored.

where the first term on the right-hand side represents the variation in the energies (and therefore light yields) of the KOs, and the second term represents the fluctuation in the number of electrons produced with energies above the Cerenkov threshold. One can show for all four sample counters that the second term is a small fraction of the total, and therefore, that variations in the energies of KOs dominate. We also note that the relative contributions to the standard error from the two sources are independent of the charge of the primary particle, since both are proportional to  $N_{\text{mat}}^{\frac{1}{2}}(Z, E, \theta)$ . Thus the fractional standard error is larger for lower charges, but the variations in the energies still dominate.

Finally, we note that the KO contribution to the total Cerenkov resolution is not dominant for typical detector parameters. Both photoelectron statistical fluctuations and the KOs contribute a fractional resolution  $\sigma_C/C \propto Z^{-1} \sec^{-\frac{1}{2}} \theta$ ; however, the former is typically two to three times larger (e.g., Grove, 1989).

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#### References:

- Berger, M. J., and S. M. Seltzer, "Table of Energy Losses and Ranges of Electrons and Positrons," NASA SP-3012 (1964).
- Grove, J. E., Ph.D. Thesis, California Institute of Technology (1989).
- Kobetich, E. J., and R. Katz, *Physical Review*, **170**, 391 (1968).
- Lezniak, J. A., *Nuclear Instruments and Methods*, **136**, 299 (1976).
- Ormes, J. F., Israel, M. H., Wiedenbeck, M. E., and Mewaldt, R. A., editors, *Astromag - The Particle Astrophysics Magnet Facility Interim Report of the Astromag definition Team*, 1986.
- Rossi, B., *High Energy Particles* (Prentice-Hall: New York, 1952).